Opredeluvanje na glavnite napreganja

Komponenti na tenzorot na napreganjeto, vo N/mm^2

$\sigma_x := -12$ $\sigma_y := 15$ $\sigma_z := 40$

$\tau_{xy} := 5$ $\tau_{yz} := 18$ $\tau_{xz} := -10$

Invarijanti na napreganjeto

$J_1 := \sigma_x + \sigma_y + \sigma_z$ $J_1 = 43$

$J_2 := \sigma_x \cdot \sigma_y + \sigma_x \cdot \sigma_z + \sigma_y \cdot \sigma_z - \tau_{xy}^2 - \tau_{xz}^2 - \tau_{yz}^2$ $J_2 = -509$

$$J_3 := \left| \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{pmatrix} \right| \qquad J_3 = -7612$$

Koeficienti na kubnata ravenka (karakteristicna ravenka)

$a := 1$ $b := -J_1$ $c := J_2$ $d := -J_3$

1. Resenie na kubnata ravenka so primena na Kardanovite formuli

$$p := \frac{1}{3 \cdot a^2} \cdot \left(3 \cdot a \cdot c - b^2\right) \qquad p = -1125.333$$

$$q := \frac{1}{27 \cdot a^3} \cdot \left(2 \cdot b^3 - 9 \cdot a \cdot b \cdot c + 27 \cdot a^2 \cdot d\right) \qquad q = -5573.074$$

$$\Delta := \frac{q^2}{4} + \frac{p^3}{27} \qquad \Delta = -4.502 \times 10^7$$ Pri Δ<0 - casus irreducibilis

$$r := \sqrt{\frac{q^2}{4} + |\Delta|} \qquad r = 7265.071 \qquad \varphi := \operatorname{atan}\left(\frac{\sqrt{|\Delta|}}{\frac{-q}{2}}\right) \qquad \varphi = 1.177$$ Modul i argument na z=α^3

$$\sqrt[3]{r} = 19.368 \qquad \sqrt{\left|\frac{p}{3}\right|} = 19.368 \qquad \text{(proverka)}$$

$k := 1..3$

$$\alpha_k := \sqrt[3]{r} \cdot \left(\cos\left(\frac{\varphi + 2 \cdot k \cdot \pi}{3}\right) + i \cdot \sin\left(\frac{\varphi + 2 \cdot k \cdot \pi}{3}\right)\right) \qquad \beta_k := \frac{-p}{3 \cdot \alpha_k}$$

$$y_k := \alpha_k + \beta_k \qquad \sigma_k := y_k - \frac{b}{3 \cdot a}$$

$\alpha_1 = -15.362 + 11.795i$ $\alpha_2 = -2.534 - 19.201i$ $\alpha_3 = 17.896 + 7.406i$

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$y_1 = -30.724$ $y_2 = -5.068$ $y_3 = 35.792$

Glavni napreganja (resenija na ravenkata od treti stepen)

$\sigma_1 = -16.39$ $\sigma_2 = 9.265$ $\sigma_3 = 50.125$

Proverka na invarijantite na napreganjeto

$\sigma_1 + \sigma_2 + \sigma_3 = 43$ $J_1 = 43$

$\sigma_1 \cdot \sigma_2 + \sigma_2 \cdot \sigma_3 + \sigma_1 \cdot \sigma_3 = -509$ $J_2 = -509$

$\sigma_1 \cdot \sigma_2 \cdot \sigma_3 = -7612$ $J_3 = -7612$

Opredeluvanje na koeficientite na pravcite na glavnite oski

$k := 1..3$

$$x_k := \frac{\left|\begin{pmatrix} -\tau_{xz} & \tau_{xy} \\ -\tau_{yz} & \sigma_y - \sigma_k \end{pmatrix}\right|}{\left|\begin{pmatrix} \sigma_x - \sigma_k & \tau_{xy} \\ \tau_{xy} & \sigma_y - \sigma_k \end{pmatrix}\right|} \qquad y_k := \frac{\left|\begin{pmatrix} \sigma_x - \sigma_k & -\tau_{xz} \\ \tau_{xy} & -\tau_{yz} \end{pmatrix}\right|}{\left|\begin{pmatrix} \sigma_x - \sigma_k & \tau_{xy} \\ \tau_{xy} & \sigma_y - \sigma_k \end{pmatrix}\right|}$$

$$\alpha_{xz_k} := \frac{1}{\sqrt{\left(x_k\right)^2 + \left(y_k\right)^2 + 1}} \qquad \alpha_{xx_k} := x_k \cdot \alpha_{xz_k} \qquad \alpha_{xy_k} := y_k \cdot \alpha_{xz_k}$$

Prva glavna oska:

$\alpha_{xx_1} = 0.921 \qquad \alpha_{xy_1} = -0.294 \qquad \alpha_{xz_1} = 0.257$

$$\frac{180}{\pi}\cdot \text{acos}\left(\alpha_{xx_1}\right) = 22.993 \quad \frac{180}{\pi}\cdot \text{acos}\left(\alpha_{xy_1}\right) = 107.102 \quad \frac{180}{\pi}\cdot \text{acos}\left(\alpha_{xz_1}\right) = 75.101$$

Vtora glavna oska:

$\alpha_{xx_2} = -0.375 \qquad \alpha_{xy_2} = -0.848 \qquad \alpha_{xz_2} = 0.374$

$$\frac{180}{\pi}\cdot \text{acos}\left(\alpha_{xx_2}\right) = 112.05 \quad \frac{180}{\pi}\cdot \text{acos}\left(\alpha_{xy_2}\right) = 147.98 \quad \frac{180}{\pi}\cdot \text{acos}\left(\alpha_{xz_2}\right) = 68.012$$

Treta glavna oska:

$\alpha_{xx_3} = -0.108 \qquad \alpha_{xy_3} = 0.441 \qquad \alpha_{xz_3} = 0.891$

$$\frac{180}{\pi}\cdot \text{acos}(-0.108) = 96.2 \quad \frac{180}{\pi}\cdot \text{acos}\left(\alpha_{xy_3}\right) = 63.82 \quad \frac{180}{\pi}\cdot \text{acos}(0.891) = 27.001$$

2. Koreni na kubnata (karakteristicnata) ravenka so pomos na funkcijata polyroot

$$\sigma_{gl} := \text{polyroots}\left(\begin{pmatrix} -J_3 \\ J_2 \\ -J_1 \\ 1 \end{pmatrix}\right) \qquad \sigma_{gl} = \begin{pmatrix} -16.39 \\ 9.265 \\ 50.125 \end{pmatrix}$$

3. Opredeluvanje na glavnite napreganja i koeficientite na pravec na glavnite oski preku sopstveni vrednosti i sopstveni vektori na matricata na tenzorot na napreganjata

$$T_\sigma := \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{pmatrix} \qquad T_\sigma = \begin{pmatrix} -12 & 5 & -10 \\ 5 & 15 & 18 \\ -10 & 18 & 40 \end{pmatrix}$$

Matrica na tenzorot na napreganjeto

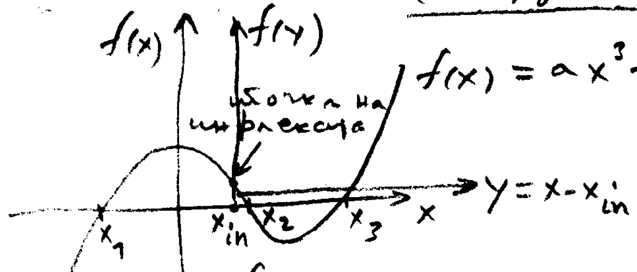
$$\sigma_{gl} := \text{eigenvals}\left(T_\sigma\right) \qquad \sigma_{gl} = \begin{pmatrix} -16.39 \\ 9.265 \\ 50.125 \end{pmatrix}$$

Glavnite napreganja se sopstveni vrednosti na Tσ

$$X := \text{eigenvecs}\left(T_\sigma\right) \qquad X = \begin{pmatrix} -0.921 & -0.375 & -0.108 \\ 0.294 & -0.848 & 0.441 \\ -0.257 & 0.374 & 0.891 \end{pmatrix}$$

Vo sekoja kolona se kosinusite na pravec na sekoja glavna oska (do na faktor +/-1). Sekoja kolona e sostven vektor na matricata Tσ

Решавање на кубна равенка
(Карданова формула



$f(x)=ax^3+bx^2+cx+d$

Со преместување на коорд. $y=x-x_{in}$, каде x_{in} е коорд. на точката на инфлексија, $f(y)$ нема да има квадратен член.

$f(x_{1,2,3})=0$

$$f''(x=x_{in})=6ax_i+2b=0 \Rightarrow x_{in}=-\frac{b}{3a}$$

$y=x-\left(-\frac{b}{3a}\right)=x+\frac{b}{3a}$, $\boxed{x=y-\frac{b}{3a}}$ - Tschirnhaus transformation

$$f(x)=a\left(y-\frac{b}{3a}\right)^3+b\left(y-\frac{b}{3a}\right)^2+c\left(y-\frac{b}{3a}\right)+d=f(y)$$

$$a\left(y^3-\cancel{3}y^2\frac{b}{\cancel{3}a}+\cancel{3}y\frac{b^2}{\cancel{9}a^2}-\frac{b^3}{27a^3}\right)+b\left(y^2-2y\frac{b}{3a}+\frac{b^2}{9a^2}\right)+cy-\frac{bc}{3a}+d=0$$

$$ay^3+0y^2+\left(c-\frac{b^2}{3a}\right)y+\frac{2b^3}{27a^2}-\frac{bc}{3a}+d=0 \;/:a$$

$$\boxed{y^3+py+q=0},\quad p=\frac{c}{a}-\frac{b^2}{3a^2},\quad q=\frac{2b^3}{27a^3}-\frac{bc}{3a^2}+\frac{d}{a}$$

Реш. на Кардано: $y=\alpha+\beta$; $\alpha,\beta=?$

$(\alpha+\beta)^3+p(\alpha+\beta)+q=0$ $\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3+p\cdot(\alpha+\beta)+q=0$

$\alpha^3+\beta^3+3\alpha\beta(\alpha+\beta)+p(\alpha+\beta)+q=0$ $\alpha^3+\beta^3+(3\alpha\beta+p)\cdot(\alpha+\beta)+q=0$

Нека $3\alpha\beta+p=0$ (1) (прв услов за α и β). Тогаш $\alpha^3+\beta^3+q=0$ (2) (втор услов)

(1): $\beta=-\frac{p}{3\alpha}$. Со смена во (2): $\alpha^3-\frac{p^3}{27\alpha^3}=-q \;/\cdot\alpha^3$

$(\alpha^3)^2+q\cdot\alpha^3-\frac{p^3}{27}=0$ $\alpha^3_{1,2}=-\frac{q}{2}\pm\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}$; $\boxed{\Delta=\frac{q^2}{4}+\frac{p^3}{27}\gtrless 0}$

(2): $\beta^3_{1,2}=-q-\alpha^3_{1,2}=-\frac{q}{2}\mp\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}=\alpha^3_{2,1}$ α^3,β^3 - можат да бидат реални или комплексни ($\Delta>0$) ($\Delta<0$)

$$y=\alpha+\beta=\sqrt[3]{\alpha^3}+\sqrt[3]{\beta^3}=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

(значи, не се важни знаците $\pm$ и $\mp$, бидејќи $\sqrt[3]{\alpha^3_{1,2}}+\sqrt[3]{\beta^3_{1,2}}=\sqrt[3]{\alpha^3_{1,2}}+\sqrt[3]{\alpha^3_{2,1}}$

Барање на $\sqrt[3]{\alpha^3}=\sqrt[3]{z}$ (сите 3 корени, од кои 2 комплексни)

$z=\alpha^3=-\frac{q}{2}+\sqrt{\underbrace{\frac{q^2}{4}+\frac{p^3}{27}}_{\Delta}}=r\cdot e^{i\varphi}$ (Ојлерова форма на комп. бр.) 2

Ако z е реално ($\Delta>0$), $r=\left|-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}\right|$, $\varphi=\begin{cases}0, & z>0\\ \pi, & z<0\end{cases}$

($u=-q/2$, $v=\sqrt{|q^2/4+p^3/27|}$)

Ако z е комплексно ($\Delta<0$), $z=u+iv$, $r=\sqrt{u^2+v^2}$, $\varphi=\mathrm{arctg}\left(\frac{v}{u}\right)$

$$\alpha_k=\sqrt[3]{z}=\sqrt[3]{r}\cdot e^{i\cdot\frac{\varphi+2k\pi}{3}}=\sqrt[3]{r}\cdot\left(\cos\frac{\varphi+2k\pi}{3}+i\cdot\sin\frac{\varphi+2k\pi}{3}\right),$$

$k=1,2,3$

Со најдени α_k, $k=1,2,3$ се наоѓаат

$$\beta_k=-\frac{p}{3\alpha_k},\quad y_k=\alpha_k+\beta_k,\quad x_k=y_k-\frac{b}{3a};\quad k=1,2,3.$$

делење со комп. број! (ако α_k е комплексно, $\beta_k=\bar{\alpha}_k$ – види ф-ла за $\beta^3_{1,2}$)

Забелешка. Ако $\Delta=\frac{q^2}{4}+\frac{p^3}{27}<0$, тогаш се јавува „casus irreducibilis" (според Кардано) и α^3 е комплексно; ($\beta_k=\bar{\alpha}_k$)

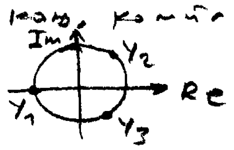
$\alpha^3=z=-\frac{q}{2}+i\sqrt{\left|\frac{q^2}{4}+\frac{p^3}{27}\right|}$. $\alpha_{1,2,3}$ и $\beta_{1,2,3}$ се конјугирано комплексни, но $y_{1,2,3}$ и $x_{1,2,3}$ се реални корени – случај на главните напрегања, деформации и мом. на инерција.

Потслучаи.

а.) $p=q=0$; $y_1=y_2=y_3=0$, $x_{1,2,3}=-b/3a$ (троен реален корен)

б.) $p=0, q\neq0$; $\alpha_{1,2,3}=0$ $\beta_{1,2,3}=\sqrt[3]{-q}$ (реален и пар кон. комп.)

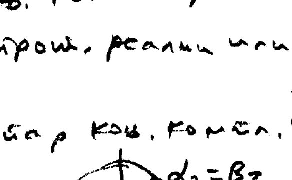
$y_{1,2,3}$ – еден реален и пар кон. комп. корени



в.) $p\neq0, q=0$; $\alpha=\sqrt{\frac{p}{3}}$, $\beta=-\sqrt{\frac{p}{3}}$ (α_1,β_1 – спр. реални или имаг., $\alpha_{2,3},\beta_{2,3}$ – кон. комп., спротивни знаци)

$y_1=0$, $y_2=\sqrt{-p}$, $y_3=-\sqrt{-p}$ (една нула и пар спрот. реални или имаг. знаци)

г.) $D=0$ и $p\neq0$; $\alpha^3=\beta^3$, $\alpha_1=\beta_1$, $\beta_2=\bar{\alpha}_2$ (1 реален и пар кон. комп. исти), $\beta_3=\bar{\alpha}_3=(\bar{\alpha}_2)=\alpha_2$



$y_1=\frac{3q}{p}$, $y_2=y_3=\frac{-3q}{2p}$ (3 реални, со еден двоен корен)

(при $\Delta>0$ сите 3 корени се реални и различни, а при $\Delta<0$ има 1 реален и 2 кон. комплексни корени)